



TAMPERE UNIVERSITY OF TECHNOLOGY
Institute of Bioelectromagnetism

Bioelectromagnetism

Exercise #3 – Answers

Q1: Measurement Lead (Lead) Vector

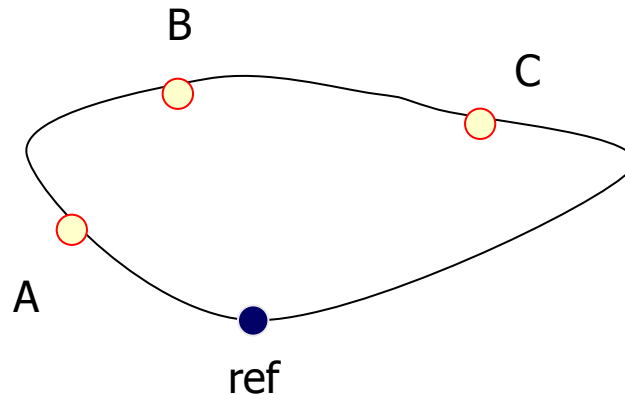
- Three electrodes (a, b and c) are on the surface of a volume conductor. Inside the conductor is a dipole source. The lead vectors of this dipole defined at the three locations (a, b and c) are:

$$\bar{c}_a = \bar{i} + 2\bar{j} + \bar{k}$$

$$\bar{c}_b = 3\bar{i} + 7\bar{j} + 2\bar{k}$$

$$\bar{c}_c = 7\bar{i} + 5\bar{j} + 4\bar{k}$$

The dipole is parallel to the unit vector \bar{i} . What is the ratio of the voltages measured between electrodes a and b to a and c?



Q1: Measurement Lead (Lead) Vector

■ \mathbf{c}_{ab} & \mathbf{c}_{ac}

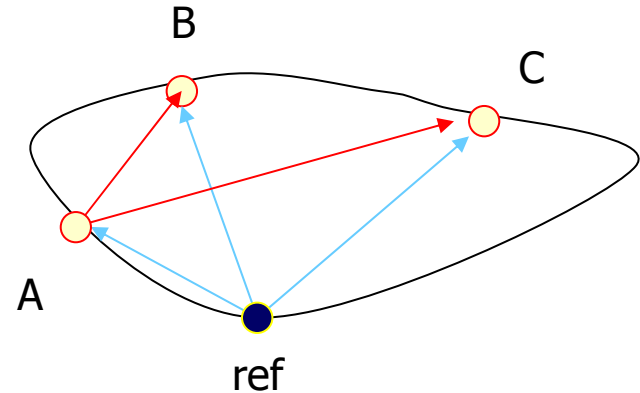
$$\bar{c}_a = \bar{i} + 2\bar{j} + \bar{k}$$

$$\bar{c}_b = 3\bar{i} + 7\bar{j} + 2\bar{k}$$

$$\bar{c}_c = 7\bar{i} + 5\bar{j} + 4\bar{k}$$

$$\mathbf{c}_{ab} = \mathbf{c}_b - \mathbf{c}_a = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$$

$$\mathbf{c}_{ac} = 6\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$$



■ V_{ab} & V_{ac}

$$\mathbf{p} = x\mathbf{i} \quad (\text{parallel to the x-axis} = \mathbf{i})$$

$$V_{ab} = 2 \cdot x \text{ [V]}$$

$$V_{ac} = 6 \cdot x \text{ [V]}$$

■ $V_{ab} / V_{ac} = 1/3$

Q2: 12-lead ECG Lead Vectors

- Derive the lead vectors of the limb leads I, II, and III, leads VR, VF, VL, and the Goldberger leads aVR, aVL ja aVF in a spherical homogeneous volume conductor.

- Preconditions

- dipole in a fixed location
- homogeneous spherical volume conductor

- Equilateral triangle

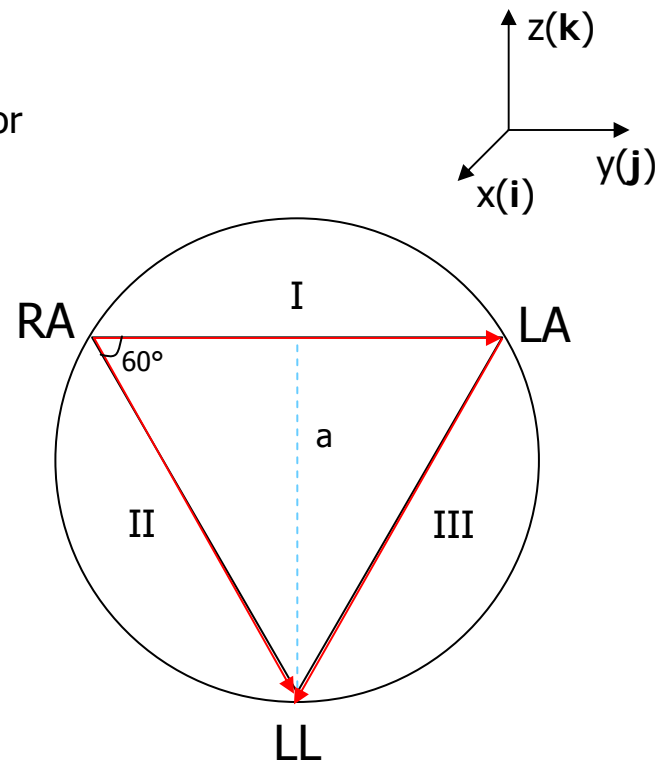
$$|C_I| = |C_{II}| = |C_{III}| (=1)$$

$$C_I = \mathbf{j}$$

$$\sin 60^\circ = |a| / |C_{II}| \implies |a| = \sin 60^\circ = |C_{II}| = \sqrt{3}/2$$

$$C_{II} = 1/2 \mathbf{j} - \sqrt{3}/2 \mathbf{k}$$

$$C_{III} = -1/2 \mathbf{j} - \sqrt{3}/2 \mathbf{k}$$



Q2: 12-lead ECG Lead Vectors

- Goldberger leads aVR, aVL ja aVF
- $|C_{aVR}| = |C_{aVF}| = |C_{aVL}| = |a| = \sqrt{3}/2$ (*|I|)

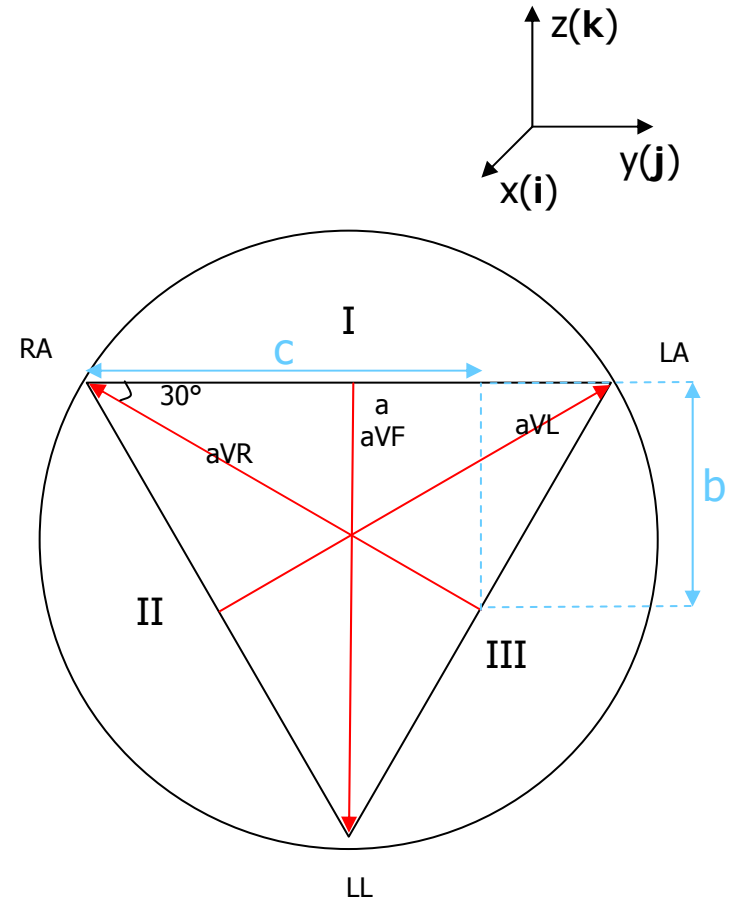
$$C_{aVF} = -|a| \mathbf{k} = -\sqrt{3}/2 \mathbf{k}$$

$$|c| = \cos 30^\circ * |aVR| = \sqrt{3}/2 * \sqrt{3}/2 = 3/4$$

$$|b| = \sin 30^\circ * |aVR| = 1/2 * \sqrt{3}/2 = \sqrt{3}/4$$

$$C_{aVL} = |c|\mathbf{j} + |b|\mathbf{k} = 3/4 \mathbf{j} + \sqrt{3}/4 \mathbf{k}$$

$$C_{aVR} = -|c|\mathbf{j} + |b|\mathbf{k} = -3/4 \mathbf{j} + \sqrt{3}/4 \mathbf{k}$$



Q2: 12-lead ECG Lead Vectors

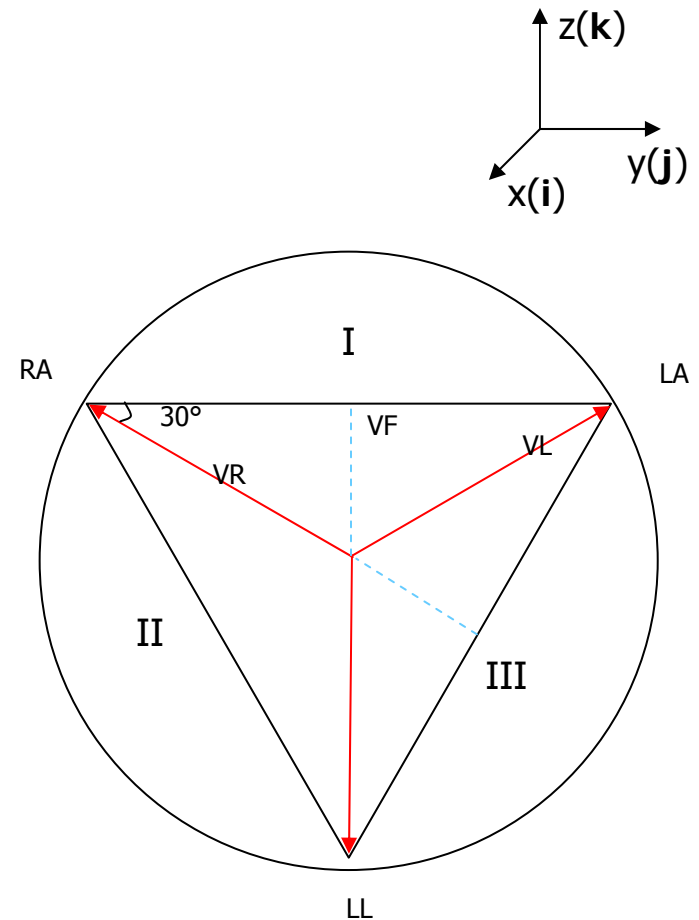
- Leads VR, VF and VL
- $|C_{VR}| = |C_{VF}| = |C_{VL}| = ?$
 $1/2 * |C_I| = |C_{VR}| * \cos 30^\circ$
 $|C_{VR}| = 1/2 * |C_I| / \cos 30^\circ$
 $= 1/2 / \sqrt{3}/2 = 1/\sqrt{3}$

$$|C_{VF}| / |C_{aVF}| = 1/\sqrt{3} / \sqrt{3}/2 = 2/3$$

$$C_{VF} = C_{aVF} * 2/3 = -\sqrt{3}/2 \mathbf{k} * 2/3 = -\sqrt{3}/3 \mathbf{k}$$

$$\begin{aligned} C_{VL} &= C_{aVL} * 2/3 = (3/4 \mathbf{j} + \sqrt{3}/4 \mathbf{k}) * 2/3 \\ &= 1/2 \mathbf{j} + \sqrt{3}/6 \mathbf{k} \end{aligned}$$

$$\begin{aligned} C_{VR} &= C_{aVR} * 2/3 = (-3/4 \mathbf{j} + \sqrt{3}/4 \mathbf{k}) * 2/3 \\ &= -1/2 \mathbf{j} + \sqrt{3}/6 \mathbf{k} \end{aligned}$$



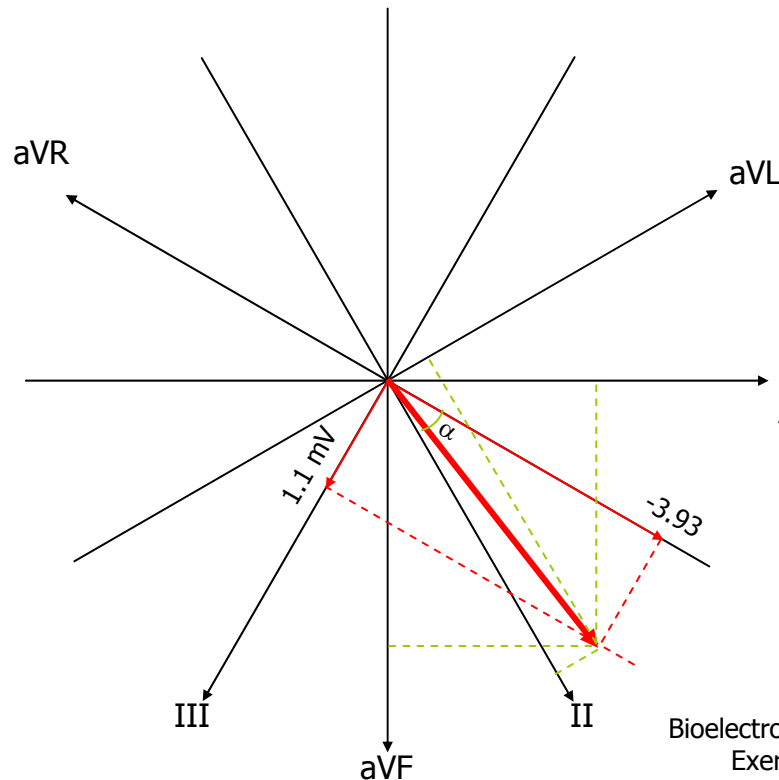
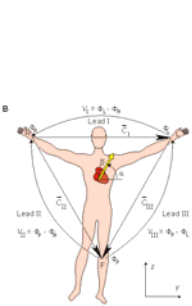
Q3: 12-lead ECG Lead Vectors

- During a QRS-complex at some time instant t the following potentials were measured:

III-lead	+1,1 mV
aVR-lead	-3,4 mV
V1 -lead	-3,5 mV

Approximate the potentials in the leads I, II, aVL, aVF, V6 and V4.

- We need \mathbf{c} & \mathbf{p} to solve the potentials ($V = \mathbf{c} \cdot \mathbf{p}$)
- Frontal plane: III & aVR identify \mathbf{p}



$$|C_I| = 2/\sqrt{3} * |C_{aVR}| \Rightarrow V'_{aVR} = -3.93 \text{ mV}$$

$$|p| = \sqrt{(3.93^2 + 1.1^2)} = 4.1 \text{ mV}$$

$$\tan \alpha = 1.1/3.93 \Rightarrow \alpha = 15.6^\circ$$

$$V_I = 4.1 * \cos(30 + \alpha) = 2.9 \text{ mV}$$

$$V_{II} = 4.1 * \cos(30 - \alpha) = 4.0 \text{ mV}$$

$$V'_{aVL} = 4.1 * \cos(60 + \alpha) = 1.0 \text{ mV}$$

$$V'_{aVF} = 4.1 * \cos(60 - \alpha) = 2.9 \text{ mV}$$

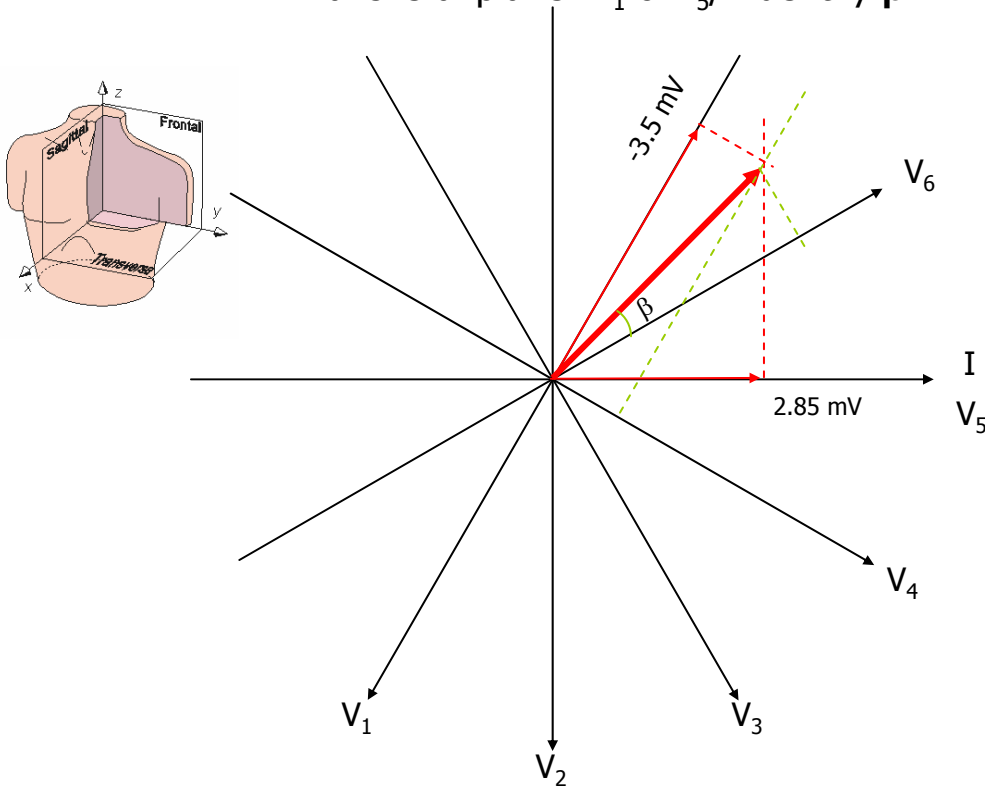
Augmented leads scaling:

$$V_{aVL} = V'_{aVL} * \sqrt{3}/2 = 0.9 \text{ mV}$$

$$V_{aVF} = V'_{aVF} * \sqrt{3}/2 = 2.5 \text{ mV}$$

Q3: 12-lead ECG Lead Vectors

- Potentials V6 and V4?
- Transveral plane: V₁ & V₅/I identify p



$$V_{V1} = -3.5 \text{ mV}$$

$$V_I = V_{V5} = 2.85 \text{ mV}$$

$$x \cdot \cos(30 - \beta) = 3.5$$

$$x \cdot \cos(30 + \beta) = 2.85$$

$$\rightarrow \beta = 10.1^\circ \text{ (HOW?, next slide)}$$

$$\rightarrow |p| = 2.8 / \cos(30 + \beta) = 3.7 \text{ mV}$$

$$V_{V6} = 3.6 \cdot \cos(\beta) = 3.7 \text{ mV}$$

$$V_{V4} = 3.6 \cdot \cos(60 + \beta) = 1.3 \text{ mV}$$

assumption: $|c_I| = |c_{V1...6}|$

Q3: 12-lead ECG Lead Vectors

Summation equation: $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$$x \cdot \cos(30 - \beta) = 3.5 \Rightarrow \cos 30 \cos \beta + \sin 30 \sin \beta = 3.5/X$$

$$x \cdot \cos(30 + \beta) = 2.85 \Rightarrow x = 2.85 / (\cos 30 \cos \beta - \sin 30 \sin \beta)$$

$$\cos 30 \cos \beta + \sin 30 \sin \beta = 3.5 / 2.85 * (\cos 30 \cos \beta - \sin 30 \sin \beta) \quad (a = 3.5 / 2.85)$$

$$\cos 30 \cos \beta - 1.25 * \cos 30 \cos \beta = -a * \sin 30 \sin \beta - \sin 30 \sin \beta$$

$$(\cos 30 - a * \cos 30) * \cos \beta = -(a * \sin 30 + \sin 30) * \sin \beta$$

$$\sin \beta / \cos \beta = -(\cos 30 - a * \cos 30) / (a * \sin 30 + \sin 30)$$

$$\tan \beta = 0.1755 \Rightarrow \beta = 10.1^\circ$$

Q4: Lead vectors...

- On the outer rim of a two dimensional volume conductor at points P_i , $i = 1, \dots, 6$ the potentials generated by a unit dipole oriented parallel to X or Y axes at point P_o are as follows:

Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5

- Using these measurements is it possible to
 - a) Derive the image surface of this source dipole location?
 - b) Calculate the lead vector of a lead between the points 3 and 6?
 - c) Derive the potential at the point 4 generated by a unit dipole at a point P_z ?
 - d) Derive the lead field of the volume conductor?
 - e) Construct a VECG lead system (X and Y-leads) that would measure the X and Y components of the dipole at the point P_o with similar sensitivity?

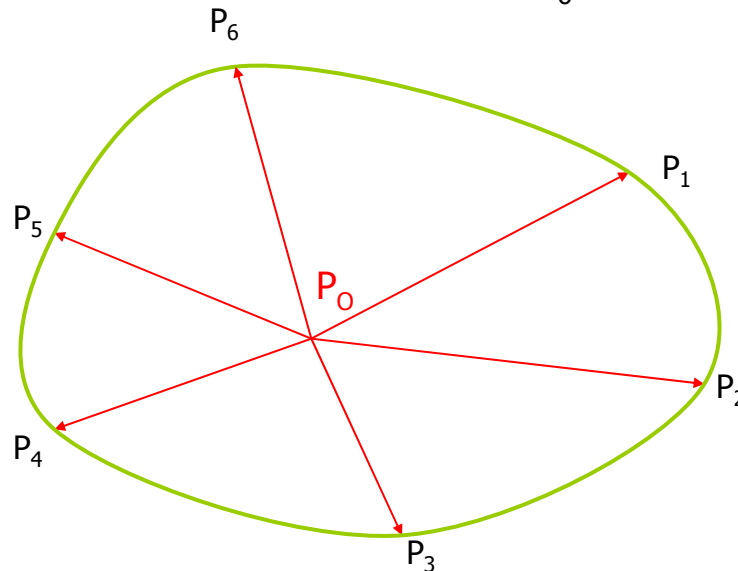
Q4: Lead vectors...

■ Is it possible to...

a) Derive the image surface of this source dipole location? yes

Electrode Potentials generated by X and Y dipoles

P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5

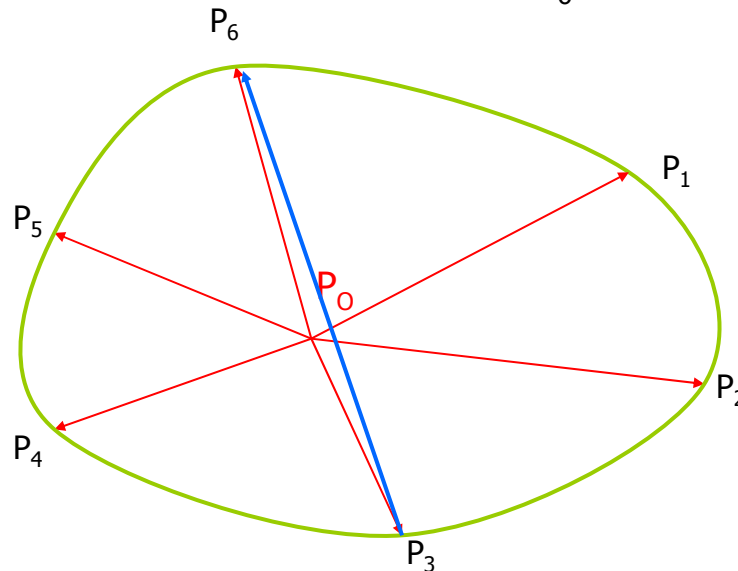


Q4: Lead vectors...

■ Is it possible to...

- b) Calculate the lead vector of a lead between the points 3 and 6?
yes

Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5

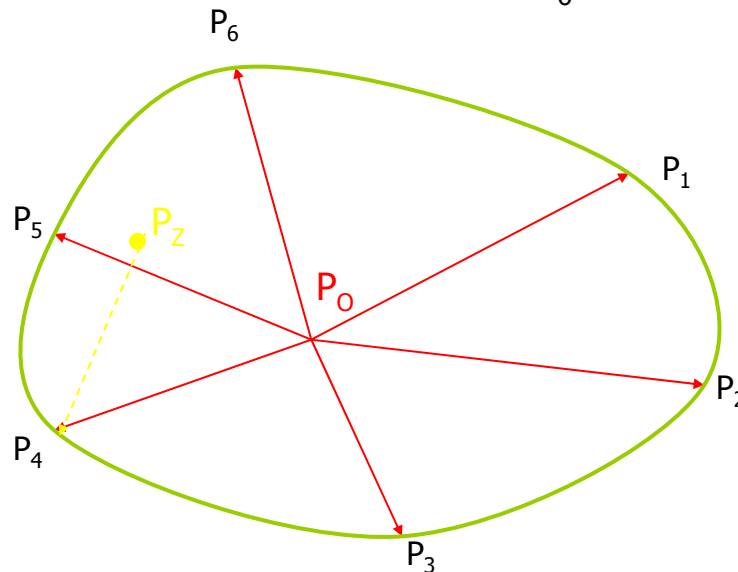


Q4: Lead vectors...

■ Is it possible to...

- c) Derive the potential at the point 4 generated by a unit dipole at a point P_z ?
no

Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5



Q4: Lead vectors...

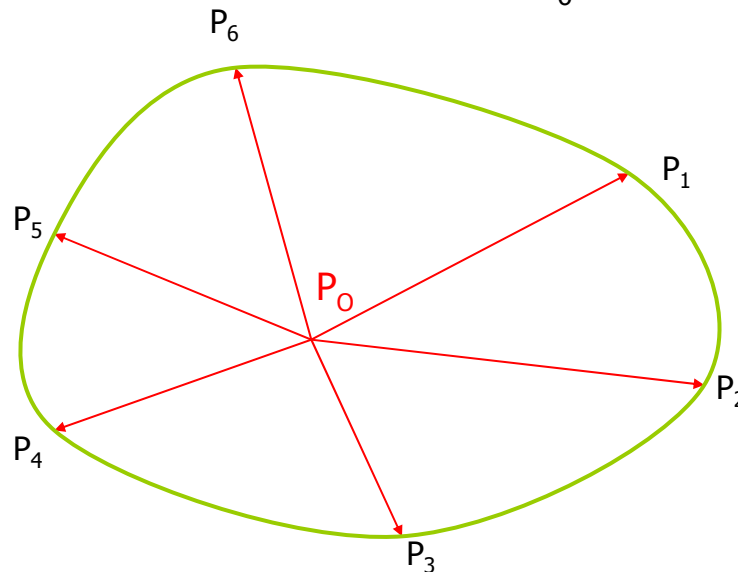
■ Is it possible to...

d) Derive the lead field of the volume conductor?

no

Electrode Potentials generated by X and Y dipoles

P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5

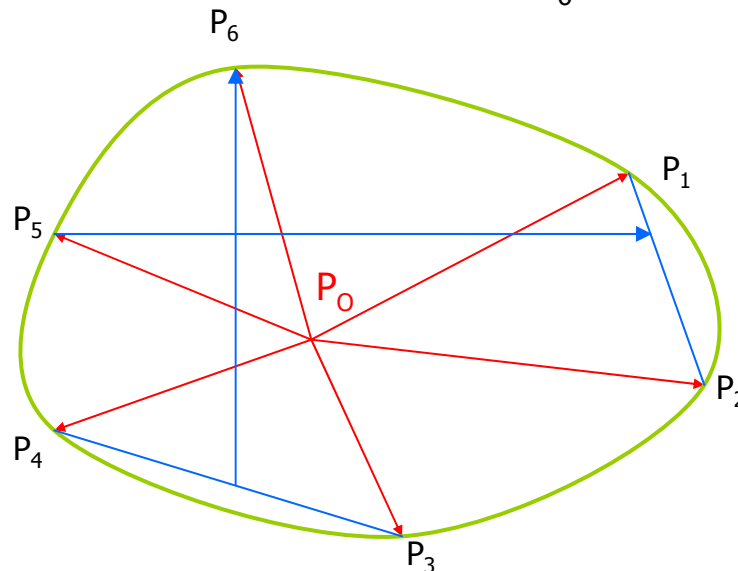


Q4: Lead vectors...

■ Is it possible to...

- e) Construct a VECG lead system (X and Y-leads) that would measure the X and Y components of the dipole at the point P_0 with similar sensitivity? yes

Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5



Q5: EEG Sensitivity

- Figure 1 represents a potential data distribution at a model of a (half) head generated by a reciprocal current of $1.0 \mu\text{A}$ applied to electrodes C and D. Calculate the potential between electrodes C and D generated by current dipoles A and B ($|P| = 4 \mu\text{Acm}$).

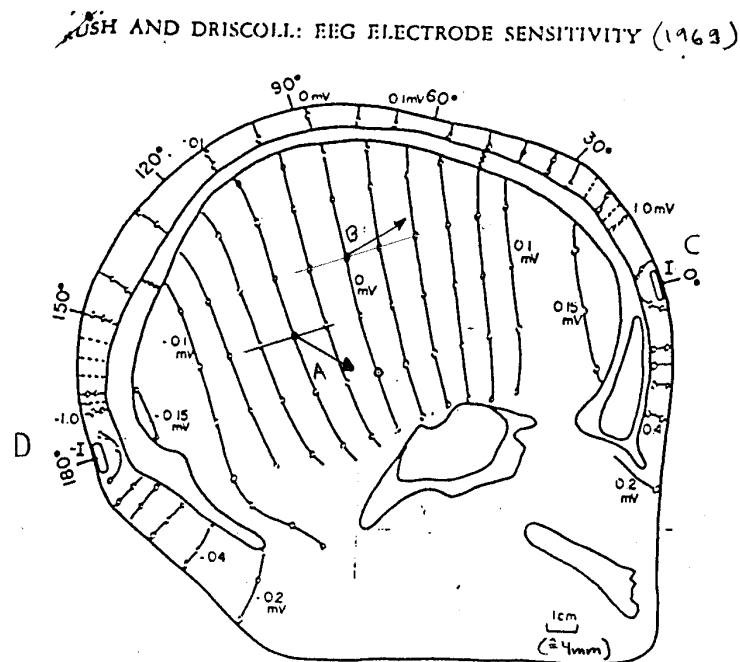


Fig 1. Potential data at surface of a electrolytic tank (midsagittal plane). Lines between 0.1 mV curves are spaced at equal potential differences (0.02 mV) apart. $I=1.0 \mu\text{A}$ in the half-head model; fluid resistivity is $2220 \Omega\text{cm}$.

Q5: EEG Sensitivity

- Reciprocal current $I_{CD} = 1.0$ μA applied to electrodes C and D forms the lead field.

- now shown as potential field
- shows the sensitivity of lead CD to dipolar sources
- distances of the lines = lead sensitivity

- "Golden equation" 11.29

$$V_{CD} = \frac{1}{I_{CD}} \int_V \frac{1}{\sigma} \bar{J}_{LECD} \cdot \bar{J}_i dv$$

reciprocal lead field source

- convert to E-field form

$$\bar{J} = \sigma \bar{E}, \quad \bar{E} = \frac{\Delta V}{\Delta x}$$

$$\Rightarrow \bar{J}_{LECD} = \sigma \frac{\Delta V}{\Delta x}$$

$$V_{CD} = \frac{1}{I_{CD}} \int_V \frac{\Delta V}{\Delta x} \cdot \bar{J}_i dv$$

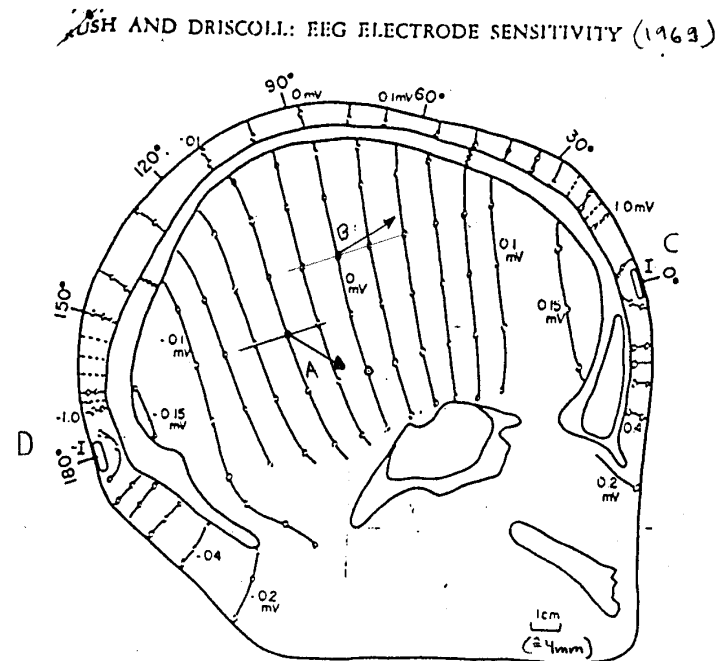


Fig 1. Potential data at surface of a electrolytic tank (midsagittal plane). Lines between 0.1 mV curves are spaced at equal potential differences (0.02 mV) apart. $I = 1.0 \mu\text{A}$ in the half-head model; fluid resistivity is $2220 \Omega\text{cm}$.

Q5: EEG Sensitivity

■ One dipole source

$$V_{CD} = \bar{c} \cdot \bar{p} = |\bar{c}| |\bar{p}| \cos \alpha$$

$$\bar{c} = \frac{\bar{E}}{I_{CD}} = \frac{\Delta V}{I_{CD} \Delta x}$$

$$\bar{p} = 4 \mu\text{Acm}$$

$$- \quad |c_A| = \frac{0.04 \text{ mV}}{2.3 \text{ cm}} * \frac{1}{2 \mu\text{A}} = 870 \frac{\Omega}{\text{m}}$$

$$\alpha \approx 43^\circ \Rightarrow V_{CD} = 25 \mu\text{V}$$

$$- \quad |c_B| = \frac{0.04 \text{ mV}}{2.45 \text{ cm}} * \frac{1}{2 \mu\text{A}} = 820 \frac{\Omega}{\text{m}}$$

$$\alpha \approx 17^\circ \Rightarrow V_{CD} = 31 \mu\text{V}$$

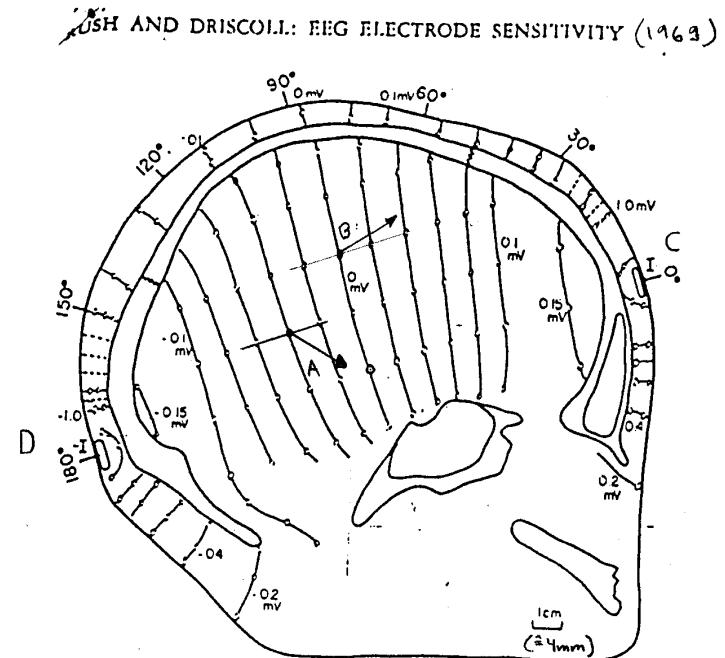


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