TAMPERE UNIVERSITY OF TECHNOLOGY
Institute of Bioelectromagnetism

Bioelectromagnetism
Exercise #1 – Answers

Q1: Equilibrium Voltages

- In nerve and muscle cells the concentration ratios of the chloride and potassium ions between intracellular and extracellular fluids are approximately 1:30 (that is, for example 4×10^{-6} : 120×10^{-6} mol/cm^3) and 38.8:1. What are the corresponding equilibrium voltages for chloride and potassium ions? (The value of the potential difference across the cell membrane that clamps the specific ionic flow)

Q1: Equilibrium Voltages

- Nernst equation separately for both ion types

$$\Delta V = -\frac{\mathrm{R}T}{z\mathrm{F}} \ln \frac{c_i}{c_o}$$

  - R = 8.314 J/(mol*K)
  - T = 37°C = 273 + 37 K = 310 K
  - F = $9.649*10^4$ C/mol

- **For Cl^-:**

$$V_{Cl} = -\frac{8.314*310}{-1*96485} \ln \frac{1}{30}$$ **= -90.9 mV**

- **For K^+ :**
  - c_i / c_o = 38.8 / 1
  - z = +1

V_K = -97.7 mV

Q2: Resting Membrane Potential

- The permeabilities of potassium, sodium and chloride ions in the giant axon of a squid at 36 °C are $P_K:P_{Na}:P_{Cl}$ = 1:0.04:0.45. The concentrations of the ions in intracellular and extracellular fluids are [10^{-3} mol/cm^3]:

$[K^+]_i = 3.45$	$[K^+]_o = 0.10$
$[Na^+]_i = 0.72$	$[Na^+]_o = 4.55$
$[Cl^-]_i = 0.61$	$[Cl^-]_o = 5.40$

Determine the resting membrane potential.

Q2: Resting Membrane Potential

- Background
  - permeability differs among ions species
  - > selective permeability -> concentration gradients - > GHK-equation
  - Nernst for one ion type. Goldman-Hodgin-Katz for several ion types:

$$V_m = -\frac{\mathrm{R}T}{\mathrm{F}} \ln\left(\frac{P_K c_{i,K} + P_{Na} c_{i,Na} + P_{Cl} c_{o,Cl}}{P_K c_{o,K} + P_{Na} c_{o,Na} + P_{Cl} c_{i,Cl}} \right) = ?$$

R = 8.314 J/(mol*K)

F = 96485 J/mol

T = 309 K

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Q2: Resting Membrane Potential

- Resting membrane potential:

$$V = -\frac{\mathrm{R}T}{\mathrm{F}} \ln\left(\frac{P_K c_{i,K} + P_{Na} c_{i,Na} + P_{Cl} c_{o,Cl}}{P_K c_{o,K} + P_{Na} c_{o,Na} + P_{Cl} c_{i,Cl}} \right) = ?$$

i) NO z
ii) == Nernst eq for K, if $c_{Na,\,Cl} = 0$
iii) equilibrium = $J_K + J_{Na} + J_{Cl} = 0$

- *RESULT* : Vm = -0.0629 V ≈ -63 mV

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Q3: Chloride Pump

- A special single cell organism that lives in a natural mineral water spring has permeabilities of 0.09, 1.00 and 0.04 for the Cl^-, K^+ and Na^+, respectively, and the following concentrations

  $[Cl^-]_i = 178$ $[Cl^-]_o = 0.47$
  $[K^+]_i = 135$ $[K^+]_o = 83$
  $[Na^+]_i = 0.05$ $[Na^+]_o = 118$

  To *maintain* the chloride gradient, this cell has a special chloride pump. Which direction does the pump have to move chloride in order to maintain the status?

- Solution: pump will have to counteract the net force (C+E)
  – GHK = resting potential
  – Nernst for Cl^-

Q3: Chloride Pump

- RMP (25 C°)

$$V = -\frac{\mathrm{R}T}{\mathrm{F}} \ln\left(\frac{P_K c_{i,K} + P_{Na} c_{i,Na} + P_{Cl} c_{o,Cl}}{P_K c_{o,K} + P_{Na} c_{o,Na} + P_{Cl} c_{i,Cl}} \right) = ?$$

$$\Rightarrow V = -6.8\,\mathrm{mV}$$

- Nernst for Cl^-:

$$V_{Cl^-} = -\frac{\mathrm{RT}}{\mathrm{zF}} \ln \frac{c_i}{c_o} = 152\,mV$$

- Driving force ΔV = RMP - V_{Cl}= -158.8 mV
  - negative ion: force to the outside
  - Cl^- leaking outside

- **Answer:**

  The pump must counteract driving force and pump chloride into the cell to maintain the chloride gradient.

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Q4: RMP

- Show that the resting potential of the cell membrane can be expressed as

$$V = \frac{g_K V_K + g_{Na} V_{Na} + g_{Cl} V_{Cl}}{g_K + g_{Na} + g_{Cl}}$$

where

g_i = membrane conductance for ions *i*,

V_i = equilibrium voltage for ions *i*.

Q4: RMP

- When the membrane is at resting state: $\Sigma\, I_i = 0$

- $I_K + I_{Na} + I_{Cl} = 0$
- $I = U/R$, $g = 1/R$

- E.g. for potassium: $I_K = g_K U = g_K (V_m - V_K)$

- For all currents:

$$g_K (V_m - V_K) + g_{Na} (V_m - V_{Na}) + g_{Cl} (V_m - V_{Cl}) = 0$$

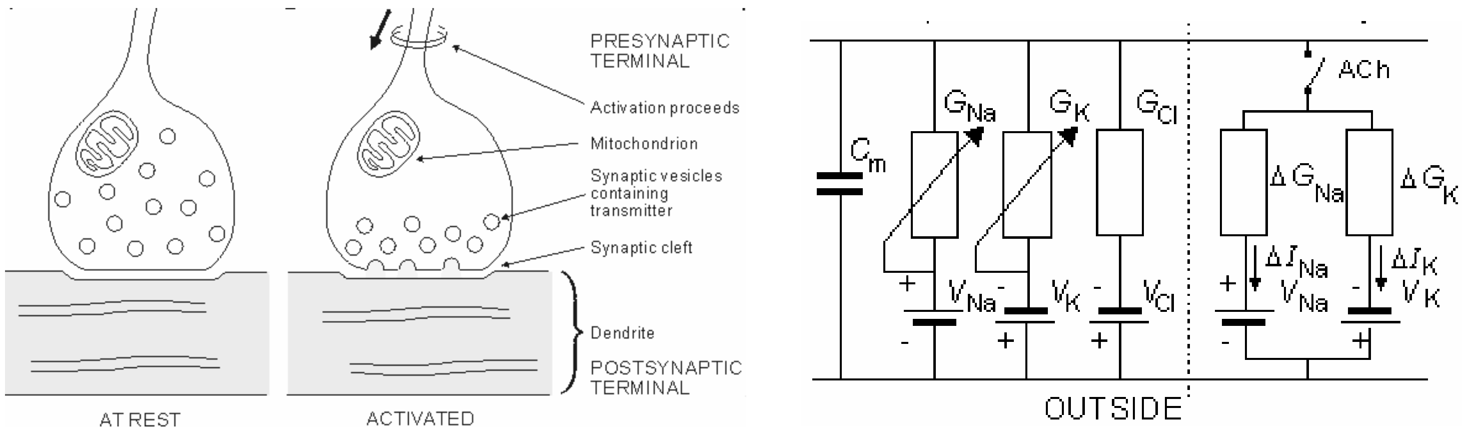
$$\Rightarrow \quad V_m = \frac{g_K V_K + g_{Na} V_{Na} + g_{Cl} V_{Cl}}{g_K + g_{Na} + g_{Cl}}$$

Q5: Synapse

- Figure 1 represents an electrical model of a synapse. When impulse arrives to the presynaptic terminal the transmitter (acetylcholine ACH) is released to the synaptic cleft (after 0.5 ms). The cell membrane will become permeable to Na+ and K+ ions and the membrane potential tends to shift towards the mean of the Nernst voltages of the Na+ and K+ ions (=reversal voltage V_{rev}).

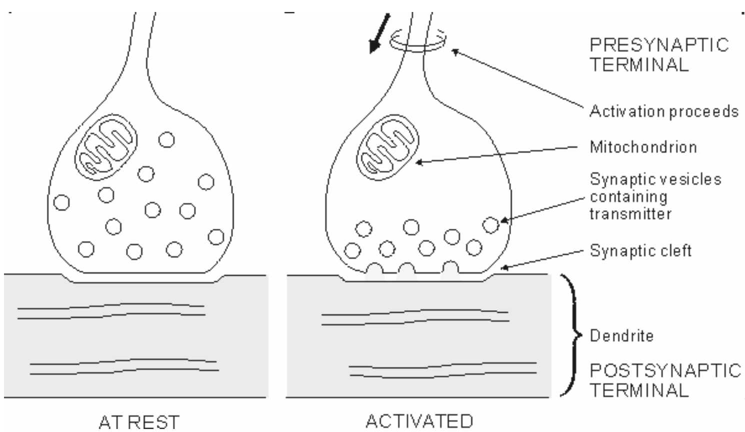
  a) What is the resting potential of the presynaptic terminal (ACH-switch open)?
  b) What is the reversal voltage, *Vrev* (ACH-switch closed)?

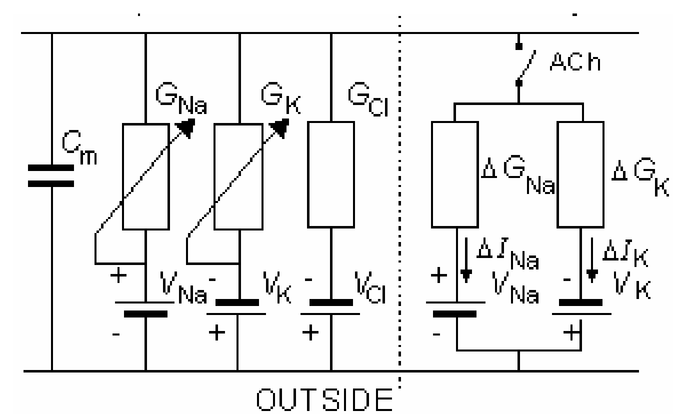
  C_m (membrane capacitance per area) = 1 μF/cm^2
  Nernst voltages V_K = -100 mV and V_{Na} = 60 mV
  $R_K = 10^8\ \Omega$, $R_{Na} = 1.5*10^9\ \Omega$, $\Delta R_K = 10^5\ \Omega$, $\Delta R_{Na} = 10^5\ \Omega$.



Q5: Synapse

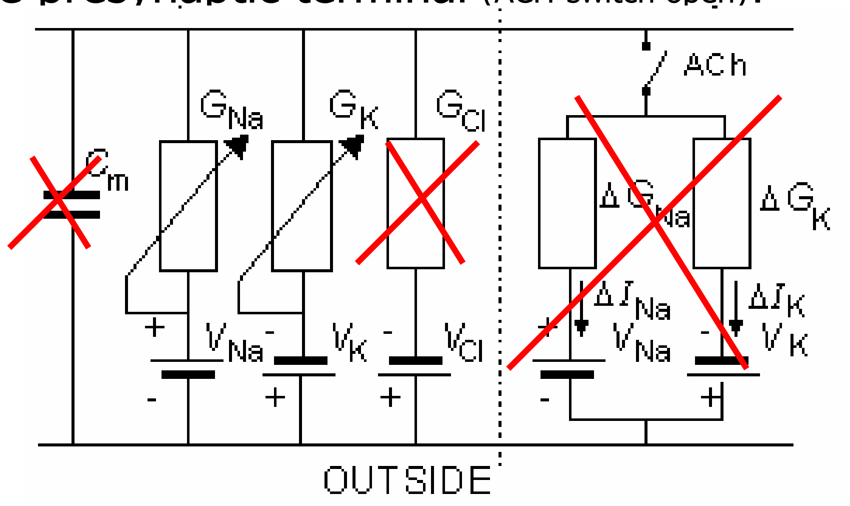
- Information from neuron to another flows across a synapse. The synapse consists of:
  - a presynaptic ending (contains neurotransmitters),
  - a postsynaptic ending that contains receptor sites for neurotransmitters and,
  - the synaptic cleft: a space between
- Neurotransmitters
  - chemicals that cross the synapse between two neurons
  - ΔGi

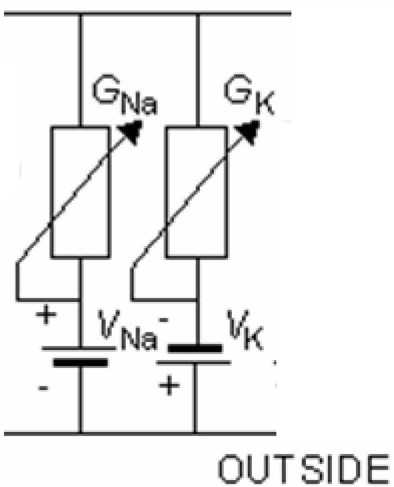




Q5: Synapse

- a) What is the resting potential of the presynaptic terminal (ACH-switch open)?
  - Ignored:
    - Na-K pumps
    - Leagage I ($\sim I_{Cl}$)
  - steady-state:
    - $i_K + i_{Na} = 0$
    - $i_K = G_K(V_m - V_K)$
    - $i_{Na} = G_{Na}(V_m - V_{Na})$
    - $G_K(V_m - V_K) = -G_{Na}(V_m - V_{Na})$
    - $V_m = (G_{Na}V_{Na} + G_K V_K)/(G_K + G_{Na})$





Q5: Synapse

Convert from R to G:

$R_K = 10^8\ \Omega \Rightarrow G_K = 1/R_K = 10^{-8}$ S

$R_{Na} = 1.5*10^9\ \Omega \Rightarrow G_{Na} = 6.7*10^{-10}$ S

$V_{Na} = 60$ mV

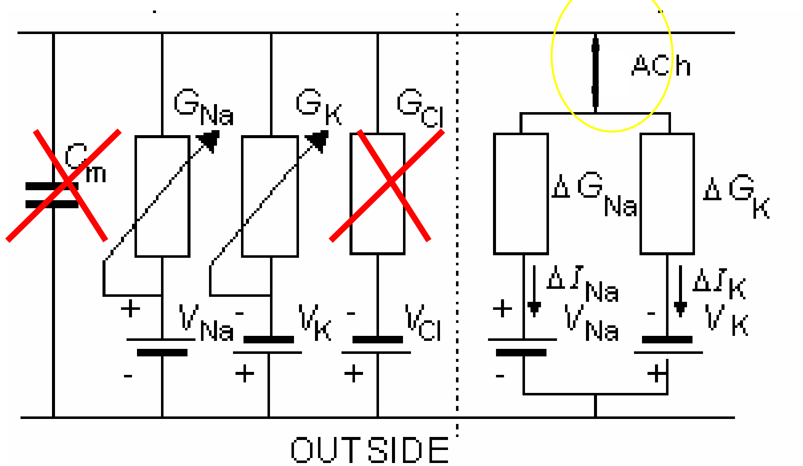
$V_K = -100$ mV

$V_m = (G_{Na}V_{Na} + G_KV_K)/(G_K+G_{Na})$

$= -90$mV

Q5: Synapse

- b) What is the reversal voltage, V_{rev} (ACH-switch closed)?
  - $\Sigma I=0$ after the release of ACH
  - $i_K+i_{Na} = 0$
  - $i_K = (G_K + \Delta G_K)*(V_r-V_K)= G'_K *(V_r-V_K)$
  - $i_{Na} = (G_{Na} + \Delta G_{Na}) *(V_r-V_{Na})= G'_{Na} *(V_r-V_{Na})$
  - $G'_K*(V_r-V_K)=- G'_{Na}*(V_r-V_{Na})$
  - $V_r = (G'_{Na}V_{Na} + G'_K V_K)/(G'_K+ G'_{Na})$



Q5: Synapse

- $V_r = (G'_{Na}V_{Na} + G'_K V_K)/(G'_K + G'_{Na})$

- $G'_{Na} = G_{Na} + \Delta G_{Na}$
- $G'_K = G_K + \Delta G_K$

- $\Delta R_K = 10^5\ \Omega \Rightarrow \Delta G_K = 10^{-5}$ S
- $\Delta R_{Na} = 10^5\ \Omega \Rightarrow \Delta G_{Na} = 10^{-5}$ S
- $G_K = 10^{-8}$ S
- $G_{Na} = 6.7*10^{-10}$ S

- $G'_K = 1.0011*10^{-5}$ S
- $G'_{Na} = 1.0001*10^{-5}$ S

- $V_{Na} = 60$ mV
- $V_K = -100$ mV

$$V_r = \frac{\frac{G'_{Na}}{G'_K} V_{Na} + V_K}{\frac{G'_{Na}}{G'_K} + 1}$$

$$\frac{G'_{Na}}{G'_K} \approx 1$$

$$V_r \approx \frac{V_{Na} + V_K}{2} \approx -20 mV$$